STUDY OF ABUNDANCE ANALYSIS OF STARS IN THE SPECTRAL RANGE B5 THROUGH G2

Grant NGR 22-024-001

Final Report

Stephen E. Strom

October 1970

Prepared for

National Aeronautics and Space Administration
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Final Report

Over the last year, considerable progress was made in attacking two problems — the blue stragglers, and the evolution of stars above the horizontal branch in globular clusters. In addition, significant progress was made in understanding the so-called super-metal-rich phenomenon in Population I K giants. A brief summary of progress in each area is included below, and attached to this final report are two preprints ("A Study of the Blue Stragglers in the Open Cluster NGC 7789" and "On the Evolutionary Status of Stars Above the Horizontal Branch in Globular Clusters").

1. The Blue Stragglers

A 2-year photometric and spectroscopic study of the blue stragglers in the galactic cluster NGC 7789 was completed at Kitt Peak National Observatory. The essential results are as follows:

A. The masses of the blue stragglers are identical with the masses of main-sequence stars at the same location in the HR diagram. This result rules out the possibility that the blue stragglers, having already evolved through the red-giant region, are on or near the main sequence for the second time in their evolutionary history.

B. Of four thoroughly studied blue stragglers, all were found to be radial-velocity variables. Although no periods were derived, these observations and independent ones by Deutsch and Peterson at the Hale Observatories provide strong evidence in favor of a binary origin for the blue stragglers. The results of this study are to be published in the November Astrophysical Journal.

2. Post Evolution of Stars above the Horizontal Branch in Globular Clusters

A spectroscopic survey of 14 stars lying above the horizontal branch in the globular clusters M3, M5, M10, M13, and M15 and NGC 6712 provided convincing evidence from radial velocities that these stars are indeed cluster members. From an evolutionary standpoint, these stars above the horizontal branch seem to divide into two groups. The first, encompassing the majority of the stars studied, lie approximately 0.5 to 1.5 mag above the horizontal branch and approximately 0.1 to 0.2 redward of the blue and of the HB of the appropriate clusters. A collaborative study with Icko Iben and Robert Rood at M.I.T. suggests that these are stars that have left the horizontal branch after exhausting helium in their cores; they are currently evolving toward the asymptotic giant branch, deriving their energy from helium and hydrogen burning in two shells. The remaining, much smaller group are considerably more luminous and have much higher effective temperatures ($L/L_{\odot}$ on the order of 1000, T_e on the order of 20,000 to 35,000 K). At least two of this group exhibit overabundances of nitrogen and oxygen and possibly of carbon. We believe these to be the descendents of stars that have ascended the giant branch for the second time. They are probably burning helium in a shell and will soon be headed for white-dwarf status when this energy source is exhausted. They appear to be analogous to the central stars in planetary nebulas. We have also found preliminary evidence in favor of mixing along the asymptotic giant branch and believe that this phenomenon is related intimately to the question of super-metallicity of Population I K giants.

3. Super-Metal-Rich K Giants

Joint work with Duane Carbon at the Smithsonian Astrophysical Observatory has provided some insight into the super-metal-rich K-giant phenomena. Analysis of 11 K giants, 5 of which were classified as super-metal-rich by Spinrad and Taylor, shows that in all but one case, the overall metal-to-hydrogen ratio in the super-metal-rich K giants is enhanced by less than 0.2 dex. The apparent line enhancements in the super-metal-rich K giants arise from a combination of enhanced microturbulence and an alteration of temperature structure near the boundary as a result of increased molecular band strengths. The details and implications of this work will be further discussed in a paper currently in preparation.

It is also noteworthy that the funds from this grant have supported the purchase of much-needed auxiliary equipment for a PDP-12 computer system, which will, within the next 6 months, provide a system for very rapid analysis of stellar spectra. The State University of New York at Stony Brook purchased a Grant microphotometer as well as the basic PDP-12 system. Software currently being developed will provide for active control of the microphotometer by the computer, very rapid tracing of most spectra (less than 5 min), and sophisticated programs for display and analysis of stellar spectra. We hope in subsequent reports to provide further details of the system, which we expect to be of considerable importance in speeding up the study of stellar spectra.

The remainder of the funds in this grant have supported several necessary trips to the Kitt Peak National Observatory by the principal investigator and K. M. Strom. These observing runs of course provided the basic material for the studies reported here.

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A STUDY OF THE BLUE STRAGGLERS IN THE OPEN CLUSTER NGC 7789

K. M. STROM*
State University of New York at Stony Brook

AND

S. E. STROM*†

* Visiting Astronomer, Kitt Peak National Observatory, which is operated by the Association of Universitites for Research in Astronomy, Inc., under contract with the National Science Foundation.

† Alfred P. Sloan Fellow.

State University of New York at Stony Brook, and Smithsonian Astrophysical Observatory, Cambridge, Massachusetts

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ABSTRACT

Image-tube spectrograms and Strömgren 4-color and Hβ photometry were obtained for twelve possible blue-straggler members of the open cluster NGC 7789. Membership was established for seven of these twelve blue-straggler candidates. From the data obtained, it was possible to test three competing hypotheses for the origin of these stars. Estimates of masses based on a match of our photometry with model atmospheres show that blue stragglers have masses appropriate to their location in the color-magnitude diagram. Furthermore, of four blue-straggler members studied carefully for radial-velocity variations, all four exhibited definite changes in velocity. These observations suggest strongly that blue stragglers are members of binary systems in which mass exchange has taken place.

I. INTRODUCTION

Several globular clusters and a few fairly old open clusters (for example, M3, M67, NGC 752, NGC 7789) contain stars which fall close to the main sequence but which extend a few magnitudes above the turnoff point of the cluster. These stars have been called "blue stragglers."

Various theoretical explanations have been offered to explain the anomalous location of the blue stragglers in the H-R diagrams. Among the most reasonable are the following:

1) The blue stragglers are representatives of a new generation of stars, born later than the majority of cluster members.

2) These stars are on (or very near) the main sequence for the second time. Possibly, a sufficient amount of the hydrogen in the envelope was transported to the interior during the helium flash to sustain a second (shorter) hydrogen-burning lifetime near the main sequence (see Rood 1970).

3) The blue stragglers are members of close-binary systems in which mass exchange has taken place (McCrea 1964; van den Heuvel 1968). On this picture, the present primary is believed to have an original mass less than the mass of stars at the turnoff, M_{to}. The mass of the *original* primary is slightly greater than M_{to}. As the original primary evolves, it fills its Roche lobe and begins to lose mass, most of which accretes on the surface of the secondary. As the secondary gains mass, it increases in luminosity and moves up the main sequence to its present location; the original primary in most cases becomes a white dwarf.

Each of these three hypotheses has one or more directly observable consequences. In hypothesis 1, the mass of the blue straggler, M_{BS}, corresponds to its location in the H-R diagram. For hypothesis 2, $M_{BS} \geq M_{to}$; whereas hypothesis 3 has the following consequences:

a) M_{BS} corresponds to its location in the H-R diagram.

b) All blue stragglers are members of binary systems, some of which should have observable variations in velocity.

c) If the initial mass of the original primary is much greater than M_{to}, it is possible to form a blue straggler; however, a straggler formed from such a system would have evolved and not be visible as such today. Hence, the blue stragglers of highest mass must have had initial mass ratios near unity, and as a consequence, $M_{BS} \lesssim 2M_{to}$. Since $M_{BS} \lesssim 2M_{to}$, the luminosity of the blue stragglers in a cluster cannot exceed the turnoff luminosity by more than a factor of approximately 10.

In order to test these competing hypotheses, we chose to study the blue stragglers in the rich open cluster NGC 7789 (Burbidge and Sandage 1958). The number and brightness of the stragglers in this cluster were consistent with our goal of obtaining fairly extensive spectroscopic and photometric observations in a reasonable time. For each star, we obtained Strömgren four-color and Hβ photometry at the Kitt Peak National Observatory. Model-atmosphere calculations can be matched with these indices to predict the luminosity, T_{eff}, and mass for the blue stragglers. In order to test the binary hypothesis, a series of spectrograms of the four brightest blue stragglers was obtained with the KPNO Casseqrain image-tube spectrograph at dispersions of 50 and 35 Å mm^{-1}. These spectra permit us to check for variations in radial velocity.

II. THE OBSERVATIONS

a) *Membership*

NGC 7789 is located in a fairly crowded region of the Milky Way ($l^{II} = 115°$, $b^{II} = -5°$). Hence, to avoid contamination from field interlopers, we felt that membership of the blue stragglers should be established on the basis of measured radial velocities. Unfortunately, to our knowledge, no published radial velocities are available for individual stars in NGC 7789.

To establish a value for the velocity of the cluster, we obtained spectrograms in the wavelength region 5000–7000 Å of K and M giants in NGC 7789 at a dispersion of 127 Å mm^{-1} with the KPNO image-tube spectrograph. The velocities were reduced by using the Grant measuring engine at the KPNO. The mean velocity deduced for three K giants is -40 ± 8 km sec^{-1}, which we adopt as the cluster velocity. In Table 1

TABLE 1

RADIAL-VELOCITY MEASUREMENTS FOR BLUE STRAGGLERS IN NGC 77–89

Star (1)	N (2)	$\langle V \rangle$ (3)	Probable Error Per Single Observation (4)	RV Member? (5)	PM Member? (6)
K144.......	1	−16	...	...	MC
K168.......	1	−21	...	...	MC
K197.......	1	−31	...	M	MC
K234.......	1	−26	...	M?	MC
K342.......	7	−45	±13	M	MC
K349.......	1	−34	...	M	MC
K371.......	4	−56	± 6	...	MC
K409.......	7	−20	±17	M?	MC
K453.......	8	−41	±19	M	MC
K799.......	12	− 2	± 7	...	NC
K1168......	1	−49	...	M?	MC
K1211......	8	−33	±18	M	MC

we list for the blue stragglers, the mean values for radial velocity, the number of observations, and the mean error per observation. These values are based on image-tube spectrograms taken at dispersions of 50 and 35 Å mm^{-1}. An M in column (5) indicates those blue stragglers which we consider to be likely members. Of the twelve stars studied, eight are probable members. We note in this regard the photometric observations of Osváth (1960): He estimates from counts of blue stars within the cluster field and in a nearby comparison field that two-thirds of the blue stars in NGC 7789 are likely to be members, which is consistent with our results. We also indicate by MC and NC, respectively, those stars thought to be members and nonmembers by Cannon (1968) on the basis of a recent, unpublished proper-motion study.

b) *The Photometry*

In Table 2, we list the observed $(b - y)$, m_1, c_1, and β for all stars in our program,

TABLE 2

STRÖMGREN FOUR COLOR AND Hβ PHOTOMETRY FOR BLUE STRAGGLERS IN NGC 7789

Star	N(Four-Color)	$(b-y)$	m_1	c_1	N(Hβ)	β
			Members			
K197	2	0.304±0.019	0.118±0.005	1.118±0.029	3	2.867±0.061
K234	2	0.303±0.027	0.173±0.054	0.938±0.049	3	2.880±0.060
K342	5	0.191±0.018	0.063±0.026	1.057±0.040	6	2.872±0.021
K409	6	0.253±0.015	0.102±0.023	1.025±0.045	8	2.930±0.049
K453	9	0.227±0.019	0.040±0.027	0.913±0.021	11	2.792±0.042
K1168	2	0.271±0.020	0.193±0.000	0.992±0.047	2	2.927±0.051
K1211	13	0.157±0.015	0.065±0.019	0.628±0.027	14	2.718±0.018
			Nonmembers			
K144	3	0.276±0.018	0.082±0.030	1.215±0.002	3	2.902±0.027
K168	2	0.258±0.021	0.119±0.041	1.060±0.043	3	2.873±0.021
K371	6	0.277±0.017	0.119±0.021	1.115±0.040	7	2.914±0.054
K799	17	0.245±0.014	0.160±0.019	0.936±0.031	17	2.867±0.034

along with the number of observations and the mean error per observation. Before comparing the photometry with model-atmosphere predications, it is necessary to correct the observed colors for the effects of interstellar reddening. From their photographic and photoelectric *UBV* observations of NGC 7789, Burbidge and Sandage (1958) derive a reddening value $E_{B-V} = 0.28$. Although these authors give no value for the expected error in this result, we estimate from their data that ±0.03 mag is a reasonable guess. We can also estimate the reddening from the espectral types derived

spectral

from our observations. In Table 3, we present for each star the estimated spectral type,

TABLE 3

REDDENING DETERMINATION FOR NGC 7789

Star	Sp Type	$(B-V)_0$	$(B-V)$	E_{B-V}
		Members		
K197..........	A3–A5 V	+0.12	0.48	+0.36
K234..........	A3–A5 V	+0.12	0.48	+0.36
K342..........	B9–A0 V	−0.03	0.30	+0.33
K349..........	A3 V	+0.09	0.43	+0.34
K409..........	A1–A2 V	+0.04	0.40	+0.36
K453..........	B9–A0 V	−0.03	0.36	+0.33
K1168.........	A3 V	+0.09	0.43	+0.32
K1211.........	B8–B9 V	−0.07	0.25	+0.32
		Nonmembers		
K144..........	B8–B9 V	−0.07	0.44	+0.51
K168..........	B9–A0 V	−0.03	0.41	0.44
K371..........	A3–A5 V	+0.12	0.45	0.33
K799..........	A2p	+0.04	0.39	0.35

the unreddened $(B - V)$ for that type (Johnson 1963), the $(B - V)$ inferred from the observed $(b - y)$ (Crawford 1966), and the resulting color excess, E_{B-V}. From this procedure we obtain $E_{B-V} = 0.34 \pm 0.01$.

Finally, we obtain a reddening estimate from the observed β and $(b - y)$ values for the stars later than A0. However, because the $[\beta, (b - y)]$-relation is still quite sensitive to gravity for stars earlier than F0, this determination is of much lower weight. A value of $E_{b-y} = 0.22 \pm 0.03$ or $E_{B-V} = 0.32 \pm 0.04$ is deduced in this way. We adopt a weighted mean value of $E_{B-V} = 0.32$ mag for the reddening to NGC 7789.

In Table 4 we present the mean values of $(b - y)$, m_1, and c_1 corrected for reddening for each star.

TABLE 4

UNREDDENED COLORS FOR BLUE STRAGGLERS IN NGC 7789

Star	$(b-y)_0$	$m_1{}^0$	$c_1{}^0$
	Members		
K197...........	+0.084	0.184	1.074
K234...........	+0.083	0.239	0.894
K342...........	−0.029	0.129	1.013
K409...........	+0.033	0.168	0.982
K453...........	+0.038	0.185	1.016
K1168..........	+0.051	0.259	0.953
K1211..........	−0.063	0.584	0.131
	Nonmembers		
K144...........	+0.056	0.148	1.171
K168...........	+0.038	0.185	1.016
K371...........	+0.052	0.185	1.071
K799...........	+0.024	0.226	0.892

TABLE 5

DISTANCE-MODULUS DETERMINATION FOR NGC 7789

Star	Sp Type	M_v	V	Apparent Modulus
		Members		
K197	A3–A5 V	+1.7	13.3	11.6
K234	A3–A5 V	+1.7	13.4	11.7
K342	B9–A0 V	+0.3	12.4	12.1
K349	A3–A5 V	+1.7	13.3	11.6
K409	A1–A2 V	+1.0	13.0	12.0
K453	B9–A0 V	+0.3	12.7	12.4
K1168	A3 V	+1.5	13.2	11.7
K1211	B8–B9 V	−0.3	11.5	11.8
		Nonmembers		
K144	B8–B9 V	−0.3	13.4	13.7
K168	B9–A0 V	+0.3	13.8	13.5
K371	A3 V	+1.5	12.9	11.4
K799	A2p	+0.8	11.8	11.0

The distance modulus is computed from the data listed in Table 5. Here we present the spectral type and luminosity class, the absolute magnitude (Blaauw 1963) corresponding to this classification, the observed V-magnitude, and the apparent modulus. From this data we obtain $(m - M) = 11.9 \pm 0.1$ (m.e.). Burbidge and Sandage estimate $(m - M) = 12.2 \pm 0.2$ from the main-sequence slope and the location of the turnoff point. We regard the agreement between these determinations as satisfactory. With $E_{B-V} = 0.32 \pm 0.03$, we predict a true distance modulus $(m - M) = 11.0 \pm 0.15$.

III. INTERPRETATION

In Figures 1, 2, and 3, we indicate the location of the blue straggler members of NGC

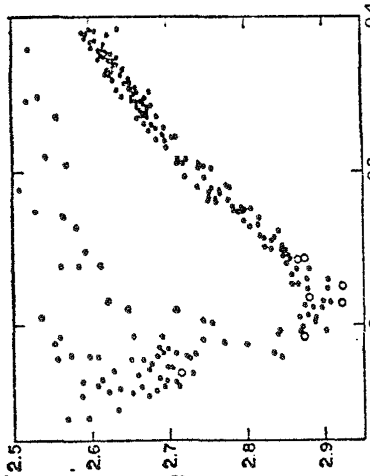

Fig. 1

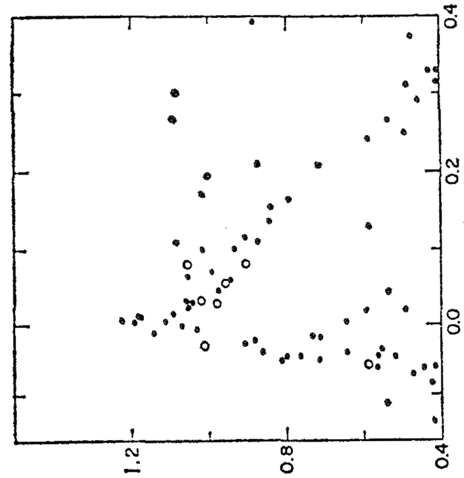

Fig. 2

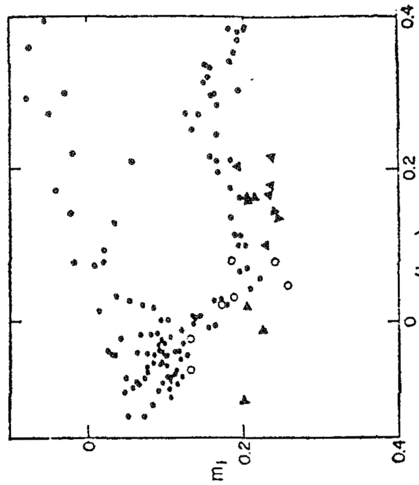

Fig. 3

Fig. 1.—Location of the blue stragglers *(open circles)* in the $[\beta, (b-y)]$-plane. For reference, the stars of luminosity classes III, IV and V as observed by Crawford (1966) are plotted. Note that, with the exception of K453, the stars fall near the lower part of the distribution, as expected for main-sequence stars.

Fig. 2.—Location of the blue stragglers in the (β, c_1)-plane. Notation is the same as in Fig. 1.

Fig. 3.—Location of the blue stragglers in the $[m_1, (b-y)]$ plane. Notation is the same as in Fig. 1 except that Am stars are plotted as filled triangles. Although K234 and K1168 appear to have high m_1 indices, the accuracy of the observations is insufficient to draw any significant conclusion as to abnormal metallicity.

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(b−y)]

7789 in the [β, ($b - y$)]-, [m_1, ($b - y$)]-, and c_1, ($b - y$]-planes. We superpose on these plots the observed location of luminosity class III-V stars as taken from Crawford's (1966) study. We note immediately that *almost without exception, the location of the blue stragglers in these plots appears to be consistent with that of normal main-sequence A and B stars*. The exceptions to this statement are (1) the location of K453 at $\beta = 2.792$ and $(b - y) = 0.038$ in the [β, ($b - y$)]-plot; the β is somewhat small for the ($b - y$); (2) the location of K1168 and possible K234 in the [m_1, ($b - y$)]-plot.

The individual observations of K453 show a range of almost 0.15 in β. This range is larger than that found for blue stragglers of comparable brightness. It is possible that the observed Hβ variation of K453 is real; and as a consequence, we have omitted this star from subsequent discussion.

Both K234 and K1168 are faint, and only two observations were obtained for each star. Since no spectral peculiarities were noted for these stars, we hesitate at this time to attribute any significance to the high m_1 values.

Although the location of the blue stragglers in the Strömgren four-color plots appears "normal," we must now interpret our observations quantitatively in order to estimate the range of stellar masses encompassed by the data. Chromey (1970) has recently computed, from model-atmosphere data, the expected variation of β with ($b - y$) and with c_1 for A- and B-type stars of differing gravities. The theoretical [β, ($b - y$)]- and (β, c_1)-plots are shown in Figures 4 and 5, in which lines of constant

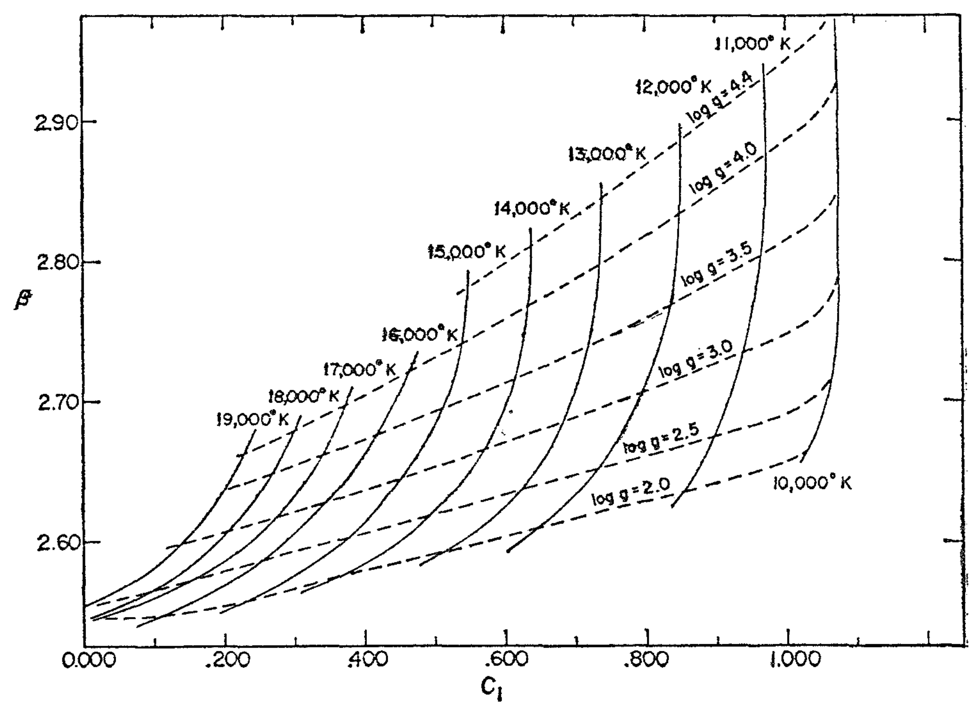

FIG. 4.—Theoretical (β, c_1)-plane as computed by Chromey (1970). Note that these plots should be used to make *differential*, as opposed to *absolute*, estimates of log g and T_{eff}.

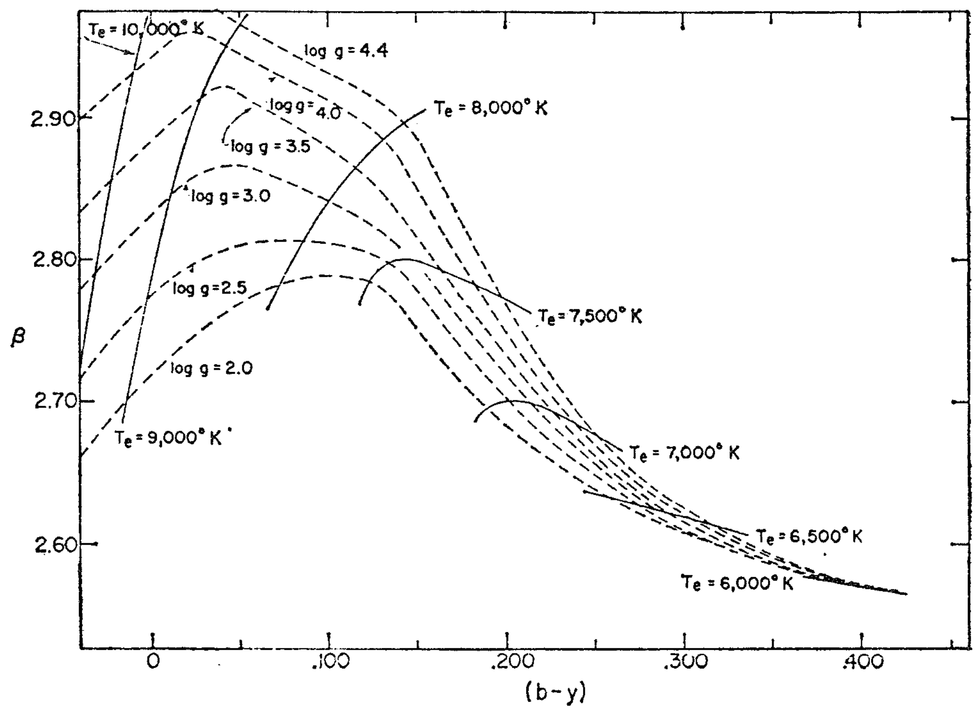

FIG. 5.—Theoretical [β, $(b - y)$]-plane as computed by Chromey (1970)

T_{eff} and log g are indicated for reference. It is essential to note that these theoretical plots are useful primarily for estimating the *differential* variation of T_{eff} and g *between* groups of stars. Because of small errors in the models and adopted line-blanketing coefficients, we caution against using these plots for absolute determinations of these parameters.

For the range of late B stars, we find from this figure that a change of 0.01 mag in β corresponds to a change of 0.1 in log g, almost independently of c_1. We can apply this result to K1211 which has a β approximately 0.05 mag smaller than the zero-age main sequence (ZAMS) value of β at $c_1 = 0.58$ (Crawford 1970, private communication). This means that K1211 has a gravity 0.5 in the log smaller than that for stars on the ZAMS near log $T_{eff} \sim 4.1$ We then estimate a value of log $g = 3.8 \pm 0.1$ for this star since B stars near the ZAMS have log $g = 4.3$ (Kelsall and Strömgren 1964; Iben 1967). From the calibration of spectral type with T_{eff} of Morton and Adams (1968), we deduce a log $T_{eff} = 4.1 \pm 0.03$ and a bolometric correction of 0.7 ± 0.2 mag. With the derived true distance modulus of $(m - M) = 11.0$, we compute the mass of K1211 as $M/M_{\odot} = 3.2\ (+1.8, -1.2)$. This mass is completely consistent with the mass of a *normal* B8 star. The luminosity at the turnoff point in NGC 7789 is log $(L/L_{\odot}) = 1.0 \pm 0.1$, while log T_e is 3.875 ± 0.015. From Iben's (1967) evolutionary tracks, the mass of stars at turnoff must be $M/M_{\odot} \sim 1.5$ with a cluster age of of $1.5 \pm 0.5 \times 10^9$ years. Thus the mass of K1211 is about twice the turnoff-point mass. As a test of the validity of our mass-determination procedure, we note that the gravity deduced from the observed Hβ index for Sirius is log $g = 4.36 \pm 0.1$ whereas the "observed" value is log $g = 4.30 \pm 0.01$. We regard this agreement in log g as excellent. The mass we deduce for Sirius is $M/M_{\odot} = 2.6$, which is entirely reasonable.

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Next, we can obtain from the [β, $(b - y)$]-diagram (for the cooler blue stragglers) estimates of log g, differential with respect to the ZAMS. From this we can again compute the mass. In Table 6, we summarize the mass determinations for the remaining blue

TABLE 6

MASS DETERMINATIONS FOR BLUE-STRAGGLER MEMBERS OF NGC 7789

Star	$[g]$	$[g/g_{\odot}]$	$[L/L_{\odot}]$	$[T_{eff}/(T_{eff})_{\odot}]$	$[M/M_{\odot}]$	$M/M_{\odot}$
K342.......	4.1±0.0	−0.35±0.1	1.84±0.1	0.242	+0.522	3.3 (+2.0, −1.2)
K409.......	4.2±0.1	−0.25±0.1	1.60±0.1	0.200	+0.55	3.5 (+2.1, −1.3)
K197.......	4.0±0.1	−0.45±0.1	1.40±0.1	0.161	+0.306	2.0 (+1.2, −0.7)
K234.......	4.1±0.1	−0.35±0.1	1.36±0.1	0.161	+0.366	2.3 (+1.4, −0.8)
K1168......	4.2±0.2	−0.25±0.2	1.44±0.2	0.181	+0.466	2.9 (+2.3, −1.3)

stragglers in NGC 7789. Brackets denote logarithms of the indicated quantities. We also plot in Figure 6 the location of the blue stragglers relative to Iben's (1967) evolu-

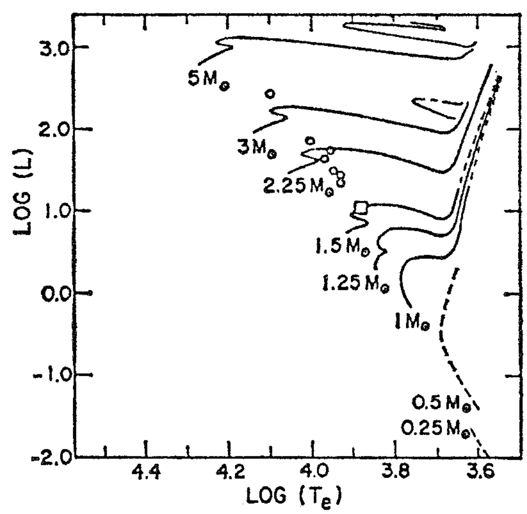

FIG. 6.—Location of the blue stragglers in the luminosity–effective temperature plane. For reference, the turnoff point of NGC 7789 is indicated as an open square. The tracks, as computed for stars of the indicated mass, are taken from Iben (1967).

tionary tracks; this figure allows us to estimate masses from the apparent location of the blue stragglers in the H-R diagram. Although no individual mass determination in Table 7 is in itself convincing, our mass determinations for the six stragglers provides compelling evidence for the statement: *The mass of blue stragglers is consistent with their observed location in the H-R diagram.* This result would appear to rule out the possibility that blue stragglers are on the main sequence for the second time.

We next turn to the question of the binary character of the blue stragglers. The essential data are summarized in Table 1. We first note that the well-observed nonmember, K799, provides one estimate of the true mean error of a radial-velocity determination from a single plate, namely, ± 7 km sec^{-1}. A further check on the accuracy of the radial velocities is afforded by a comparison of our image-tube radial velocities with those derived by Trumpler (1930) for eleven B stars in NGC 2264. The comparison gives $(V_{\text{Strom}} - V_{\text{Trumpler}}) = -8 \pm 6$ km sec^{-1}; the error refers to the accuracy of an individual observation. Finally, we note that for five G-star members of M67, $(V_{\text{Strom}} - V_{\text{true}}) = 0 \pm 4$ km sec^{-1}. Hence, we believe that ± 7 km sec^{-1} is a conservative estimate of our *internal* error for radial-velocity determinations. It is clear immediately from Table 1 that the four well-observed blue stragglers have significantly higher errors per individual velocity determination. The most likely explanation for these greater errors is that these stars have variable radial velocities. While the velocity variations do not provide definite proof of the binary character of the blue stragglers, our data suggest strongly that *all blue stragglers studied are probably spectroscopic binaries.* We further expect the rotational velocities of the blue stragglers to be relatively small as a consequence of mass exchange and the close coupling between rotational and orbital angular momentum (van den Heuvel 1968) in the binary system. All blue stragglers in NGC 7789, with the exception of K453, have $v \sin i$ M 100 km sec^{-1}, that is, less than the resolution of the image-tube spectrograms at 50 Å mm^{-1}. Spectrograms of K121, K409, and K342 were obtained at 35 Å mm^{-1}; and we estimate in these cases that $v \sin i \lesssim 70$ km sec^{-1}. The Ca *II* K-line and hydrogen-line widths in K453 appear to be variable on our spectrograms, as might be expected from the results of our Hβ photometry. This star may be a double-line binary; however, confirmation of this suspicion must await further observation. We conclude that none of the blue stragglers have rotational velocities greater than can be determined at our resolution. This observation is again consistent with the binary hypothesis.

To test whether we *expect* on the basis of the binary hypothesis to observe velocity variations for most blue stragglers in NGC 7789, we chose the most pessimistic case, that of K1211. From its location in the H-R diagram, we note that it is the most massive of the blue stragglers. Hence, its companion must presently be the *least* massive. As a consequence, we expect the observed variations in velocity for this star to be a minimum. As a plausible model for the initial configuration, we take $m_1{}^0 = 1.7\ M_\odot$ and $m_2{}^0 = 1.5_\odot$. For the final configuration, $m_1 = 0.5\ M_\odot$ and $m_2 = 2.7\ M_\odot$. Defining α as m_2/m_1, we can compute the ratio of the present to the initial period from (see van den Heuvel 1968) the equation

$$P/P_0 = \{\alpha_0/\alpha[(1+\alpha)/(1+\alpha_0)]^2\}^3 .$$

Adopting the above parameters, we obtain $P/P_0 \sim 6.7$. The total velocity amplitude, $2K_1$ in km sec^{-1}, is given by

$$2K_1 = \frac{208(m_1 + m_2)^{1/3} \sin i}{P^{1/3}(m_2/m_1)^{1/2}} ,$$

where i is the angle of inclination. If we adopt a mean value of $\sin i$ of 0.7, we find that for our limit of detection ($2K_1 \gtrsim 30$ km sec^{-1}), the present period P is less than about 27 days. Hence, the *initial* period of the binary must have been $P_0 \lesssim 4$ days. Approximately 25 percent of field binaries have periods in this range (Abt 1961). As an extreme example (see Refsdal and Weigert 1969) we choose $m^1{}_0 = 1.7\ M_\odot$ and $m_2{}^0 = 1.5\ M_\odot$; $m_1 = 0.3\ M_\odot$ and $m_2 = 2.9\ M_\odot$. For this case, the present period must be less than ~ 13 days. For this pessimistic case, only a few percent of the initial binaries would be expected to have periods in this range. It would appear that, unless extreme conditions prevail, it is quite likely that a blue-straggler system similar to K1211 will be detected given our sample and our observational accuracy.

We can also check to see that the number of blue stragglers is consistent with the expected number of progenitor binary systems. We note first that all blue stragglers formed from systems with an initial primary mass greater than $\sim$1.7–1.8 $M_\odot$ would have evolved already and would now be in a later stage of evolution. Second, from the observed luminosities as well as our deduced masses, we note that the minimum mass of the blue stragglers is $\sim 2\ M_\odot$. On the assumption that a star of $1.7\ M_\odot$ is not likely to lose more than $1.4\ M_\odot$, the initial mass for the original secondary must be $0.\geq 6\ M_\odot$. Hence, the initial mass ratio must be in the range $1.0 \leq a \leq 3$. Approximately half the total number of binaries in NGC 7789 should have mass ratios in this range (van den Heuvel 1968). Furthermore, about half of these systems have appropriate initial separations (i.e., periods) for mass exchange to occur. If binary frequency in NGC 7789 is similar to that in the Hyades, approximately 25 percent of all stars are binaries. Consequently, 6 percent of all NGC 7789 members from the turnoff to approximately 2 mag ($M/M_\odot \sim 0.6$) below should form blue-straggler systems. If the survey of Burbidge and Sandage is reasonably complete, there are $\sim$330 stars occupying this region of the H-R diagram; as a result, we would expect approximately twenty blue stragglers. However, we estimate, using the luminosity function given in Allen (1963), that approximately 50 to 100 stars below the turnoff are probably field interlopers. This reduces the expected number of blue stragglers to the range 14–17. We observe eight blue stragglers to be certain members. Since we have not spectroscopically tested fourteen possible blue stragglers for membership, the total number must be in the range $8 < N < 21$. We consider this comparison to be reasonably consistent with the number of blue stragglers expected.

Finally, we note that Deutsch (1969) and Peterson (1970) have examined several stragglers in the open cluster M67. One of these stars has a definite period, while three others are radial-velocity variables. Hence, it appears reasonable to suggest that *most, if not all, blue stragglers are members of binary systems.*

IV. CONCLUSIONS

From our investigation we find that:

1. The masses deduced for blue stragglers from Strömgren four-color and Hβ photometry are consistent with their location in the H-R diagram.
2. All blue stragglers studied for radial velocity are velocity variables.

Conclusion 1 rules out the possibility that blue stragglers are on the main sequence for a second time during their evolutionary history. Conclusion 2 permits the following alternates:

a) The blue stragglers are a class of newly formed main-sequence stars, all members of which are binaries and all of which have relatively low rotational velocities.

b) The blue stragglers are produced as a result of mass exchange in close binaries. We believe the second of these alternatives to be the more likely.

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REFERENCES

Abt, H. A. 1961, *Ap. J. Suppl.*, **6**, 37.
Allen, C. W. 1963, *Astrophysical Quantities* (London: Athlone Press).
Blaauw, A. 1963, in *Basic Astronomical Data*, ed. K. Aa. Strand (Chicago: University of Chicago Press).
Burbidge, E. M., and Sandage, A. R. 1958, *Ap. J.*, **128**, 174.
Cannon, R. D. 1968, private communication.
Chromey, F. R. 1970, private communication.
Crawford, D. L. 1966, *I.A.U. Symposium No. 24*, p. 170.
Deutsch, A. J. 1969, private communication.
Heuvel, E. P. van den. 1968, *B.A.N.*, **19**, 326.
Iben → Iken, I. Jr. 1967, *Ann. Rev. Astr. and Ap.*,
Johnson, H. L. 1963, in *Basic Astronomical Data*, ed. K. Aa. Strand (Chicago: University of Chicago Press).
Kelsall, T., and Strömgren, B. 1964, *Vistas in Astronomy*, Vol. 8, ed. A. Beer and K. Aa. Strand (Oxford: Pergamon Press).
McCrea, W. H. 1964, *M.N.R.A.S.*, **128**, 147.
Morton, D. C., and Adams, T. F. 1968, *Ap. J.*, **151**, 611.
Osváth, I. 1960, *Abh. Hamburg-Bergdorf*, **5**, 129.
Peterson, D. M. 1970, private communication.
Refsdal, S., and Weigert, A. 1969, *Astr. and Ap.*, **1**, 167.
Rood, R. T. 1970, *Ap. J.* (in press).
Trumpler, R. J. 1930, *Lick Obs. Bull.*, **14**, 154.

Note added in proof:

Spectra of four additional K-giants were obtained subsequent to the acceptance of this paper. The cluster velocity for NGC 7789 derived from a total of 7 K-giants is -45 ± 7 km/sec. As a consequence of this newly derived velocity, we now believe K371 to be a probable member. The photometry of this star predicts a mass consistent with its location along the main sequence, entirely in keeping with the conclusion of this paper.

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On the Evolutionary Status of Stars Above the Horizontal Branch in Globular Clusters*

S. E. Strom** ***
State University of New York at Stony Brook
and
Smithsonian Astrophysical Observatory, Cambridge, Mass.

K. M. Strom**
State University of New York at Stony Brook

R. T. Rood and I. Iben, Jr.
Massachusetts Institute of Technology

Received May 4, 1970

Spectra of 13 stars falling above the horizontal branch in the globular cluster M2, M10, M13, and M15 and NGC 6712 were obtained with the Carnegie image tube spectrograph at the Kitt Peak National Observatory. At least 11 and probably all of these stars are shown to be members on the basis of radial velocity measurements. Of this group, 3 stars are hot enough to show lines of He I, and to within the estimated errors, the abundance of helium is approximately the same as for Pop I B-stars. These three stars are very luminous and probably burning helium in a shell after a second ascent of the giant branch; that is, they are probably similar in evolutionary state to the cores of planetary nebulae. There is some evidence that in at least two of these stars (M10 1 – 33 and Von Zeipel 1128 in M3) significant mixing between the surface and the interior has altered the surface composition. It is suggested that the remaining 8 stars have left the horizontal branch and are burning helium in one shell and hydrogen in another. These 8 stars are most likely the progeny of stars which occupied the very blue end of the horizontal branch. The latter are presumed to have lost a significant amount of mass. The evolutionary lifetimes of these stars above the horizontal branch are consistent with this suggestion for their origin.

Key words: [redacted]

I. Introduction

Most of the classical features of globular cluster diagrams have already been related to the properties of theoretical models. The main giant branch has for many years been known as the residence of stars that are burning hydrogen in a shell outside of an inert helium core. Many features of the horizontal branch may be accounted for by models which are burning helium at the center and hydrogen in a shell. Finally, stars on the asymptotic giant branch can be accounted for in terms of models that are burning helium and hydrogen in two separate shells outside of an inert carbon-oxygen core.

In contrast, the evolutionary status and indeed the cluster membership of stars lying above the horizontal branch and far to the blue of the main giant branch in globular clusters has long been ignored. Although their total number is not negligible, these stars are spread out in the color magnitude diagram at such a low density that no clear cut pattern is discernable.

As yet, no theoretical models have been published that pass at the appropriate rates through all of the regions occupied by: (1) bright blue stars such as Von Zeipel (VZ) 1128 in M3; (2) W Virginis and BL Her stars that might seem to define a sequence to the blue of the asymptotic branch; and (3) stars that are clearly not a part of the horizontal branch, but are scattered seemingly at random in $B-V$ at about a magnitude or so above the horizontal branch. Are all of these stars in stages more advanced than those DSS (double shell source) models that account for the asymptotic branch?

It has been argued before, on the basis of surface composition anomalies as well as on the basis of color and magnitude, that the brightest blue stars, as typified by VZ 1128, are indeed in a stage more advanced than the DSS models. We concur with these arguments.

On the other hand ,we believe that many of the other stars are in the DSS stage, being a part of a large diffuse grouping that includes the asymptotic giant branch stars. We suggest that, in addition to the time it has already spent in the DSS stage, one of the major parameters that distinguishes one star from another in this group is its total mass.

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II. Cluster Membership and Surface Composition

Before embarking on a model study to explain them, it is advisable to ensure that at least a sizeable fraction of the stars that *appear* to lie above the horizontal branch are indeed cluster members rather than field interlopers. Both in order to establish membership and to identify possibly unusual composition features that might be expected at the surfaces of highly evolved stars, two of us (S^2) undertook a spectroscopic survey of stars in M2, M10, M13, M15, and NGC 6712.

Thirteen stars located from 0.5 to 3.5 magnitudes above the horizontal branch were selected for study from Arp's (1955) survey of seven globular clusters. In addition, the bright star C26, in NGC 6712 (Sandage and Smith, 1966) was chosen because of its relative accessibility to spectroscopic analysis. The location in the color-magnitude diagram of five of the program stars in M13 is shown in Fig. 1. Spectra with a dispersion of 50 Å/mm were obtained at the Kitt Peak National Observatory using the Carnegie image tube spectrograph. Typically, these spectra were widened to 0.2 or 0.4 mm and covered the wavelength range 3600 to 4900 Å. A spectrum of a star with $V = 15$ and $B-V = 0$ widened to 0.4 mm required an exposure time of approximately 90 minutes. Great care was taken to record the contribution of sky and cluster background on either side of the stellar spectrogram; however, in all cases these contributions were insignificant. A radial velocity was obtained for each spectrogram from measurements made with the Grant measuring engine at the Kitt Peak National Observatory; the error per individual velocity determination is estimated to be ± 9 km/s (p.e.). A summary of our deduced radial velocities for each star appears in Table 1. Also indicated in Table 1 is the cluster velocity as determined by Mayall (1946) and by Kinman (1959); the error in the determination of a cluster velocity is estimated to be approximately ± 10 km/s.

In all cases except those of M2 and M10, the cluster velocity is sufficiently different from the velocities expected for foreground stars that membership can be established unambiguously. For M13, the case for membership is further strengthened by the availability of proper motions (Kadla, 1964). In all cases radial velocities and proper motions indicate that our program stars are members. For M2 and M10 we estimate, from the expected density of stars above the plane, that the probability of finding an intervening A or B star of the appropriate apparent magnitude within 5 arc minutes of the cluster center is less than 10^{-2} (Allen, 1963). Since the stars observed lie within 5′ of the cluster center for both M2 and

M 10 and, moreover, since the velocities are in good agreement with the cluster values, we feel relatively confident that these stars are indeed cluster members. In Table 2 we present data which locates each of our program stars relative to the appropriate cluster horizontal branch. The cluster and star names are given in columns 1 and 2; in column 3 is the visual magnitude; in column 4 is the difference in V magnitude between the program star and the horizontal branch at the $(B-V)$ color of the program star; in column 5 is an estimated difference in $(B-V)$ between the blue end of the horizontal branch and the location of the program star.

We give in Table 3 a description of the spectrum for each of the stars on our program. We note immediately that four of these stars, Barnard 29, Von Zeipel 1128, M 10 I, 33 and NGC 6712 C26 show evidence for helium lines in their spectra. Previous analyses of Barnard 29 and VZ 1128 indicate that the He line strengths in these stars are consistent with a Pop I abundance.

For M 10 I, 33 no photoelectric colors are available so that it is difficult to estimate its T_{eff}; consequently the He abundance estimate is uncertain. If we adopt $(B-V) = +0.20$ as the color at the blue edge of the RR Lyrae gap in M 10, Arp's (1955) photographic photometry suggests $(B-V)_0 \sim -0.25 \pm 0.05$ for M 10 I, 33. This color implies a spectral type in the range B0 to B3, although the appearance of the He I singlets and triplets implies a type closer to B5. In Table 4 we present the equivalent widths derived from our spectrograms of M 10 I, 33 along with those measured by Wright *et al.* (1964) for ι Her (B3V). The estimated error for our equivalent width values is $\pm 20\%$. Since the He I lines reach maximum strength near type B3, it would appear from this table that the He abundance in M 10 I, 33 is certainly very close to Pop I values. A more exact statement regarding the He abundance awaits determination of more accurate colors.

In the case of NGC 6712 C26, we note that the unreddened color is $(B-V)_0 = -0.06$, which suggests a spectral type of about B8–B9. The helium line strengths shown by Pop I B8–B9 stars are typically 0.2 to 0.3 Å. For image tube spectra taken at our dispersion, the limiting equivalent widths detectable are about 0.15 to 0.2 Å. Since we are able to detect He I λ3820, 4026 and 4471 in C 26, we estimate that the He abundance for this star is very close to Pop I values. No other stars in our observing program are hot enough to show He lines.

At first glance it appears that at least some globular cluster stars provide direct spectroscopic evidence in favor of a high initial He content. Hence, one may be tempted to attribute the low helium content deduced for several BHB stars (Searle and Rodgers, 1966, Sargent and Searle, 1966; Greenstein and Münch, 1966) to gravitational settling (Greenstein *et al.*, 1966), or to some as yet unknown mechanism which radically alters the atmospheric composition relative to the interior composition. For Von Zeipel 1128, the spectrum suggests the possibility of Population I (or greater) abundances for O and N and, with less certainty, for Si. In contrast, the cluster metal to hydrogen ratios are lower by a factor of 10 to 20 than typical Pop I ratios. On the basis of this contrast, Strom and Strom (1970) argue that Von Zeipel 1128 has very likely undergone extensive mixing during its lifetime, possibly during the late DSS phase of its evolutionary history.

It would appear from Table 3 that M 10 I, 22 is quite similar in character to VZ 1128. The halo field star HD 214539 (A2Ib) also appears to exhibit a Pop I helium content but an unusual distribution of heavy element abundances (Przybylski, 1969). From its high luminosity and unusual composition we suggest that HD 214539 may be an analogue of VZ 1128.

For NGC 6712 C26, we would be unable to detect all but very large enhancements in light element abundances. At the effective temperature values appropriate to type B8, lines of C, N and O cannot be seen at our dispersions even if these elements are present at Pop I abundances. In terms of its composition, Barnard 29 seems to be the exception in our group of hot, blue stars above the horizontal branch that appear to have a Pop I helium abundance and also an unusually high abundance of O and N relative to metals. Although it apparently has a normal (Pop I) helium abundance, Barnard 29 has no obvious abundance anomalies among the heavier elements (Stoeckley and Greenstein, 1969).

Examination of the spectra of the other stars in our program provides no surprises. All of the spectra appear to be similar to those of weak line horizontal branch stars. The hydrogen lines are in most cases unusually sharp, no doubt owing to low surface gravities ($\log g \sim 1.5$ to 2.0).

We also searched the spectra of all of our program stars for any evidence of enhanced abundances for s-process elements. We found none. Unfortunately, the luminous stars observed in our program have T_{eff} values too great to permit observation of s-process element spectral lines in the visible region of the spectrum.

In summary, all of our 13 program stars appear to be cluster members. The bright, blue stars that apparently have a normal (Pop I) surface helium abundance all lie more than 2 magnitudes above the horizontal branch (see Table 2). With the exception of Barnard 29, these stars also seem to have abnormally large abundances of O and N (and possibly Si) relative to the metal abundances of other cluster stars. All of the other stars in our sample lie between 0.8 and 1.5 magnitudes above the horizontal branch and are at least 0.15 mag redder in $(B-V)$ than the blue end of the horizontal branch. The spectra for all of the stars in this latter group are nearly indistinguishable from the spectra of weak line horizontal branch stars.

III. On the Location of Double Shell Source Models

Having been convinced of the existence of a cluster feature above the horizontal branch, two of us (RI) constructed a set of DSS models for stellar masses somewhat lower than those necessary for describing an asymptotic branch. The input physics for the models has been described in Iben and Rood (1970a) (Cox-Stewart opacities etc.). Each evolutionary track was started at the point where the star first arrived on the horizontal branch. The core helium burning phase and the double shell source phase for some of the models have already been discussed in Iben and Rood (1970b).

The helium abundance parameter Y and the metal abundance parameter Z have been chosen on the basis of comparisons between model properties and the classical features of cluster diagrams (e.g., Iben and Rood, 1969; Iben, Rood, Strom and Strom 1969). The spread in the masses chosen for initial models has been dictated by the fact that the spread in color along observed horizontal branches (Newell, 1970) can not be matched by the evolutionary track of a single model star of a given mass (Iben and Rood, 1970b). Iben and Rood find that a spread in masses on the order of 20 precent is necessary to account for the distribution in color along the horizontal branch of clusters such as M3 and M15.

In Fig. 2, the dashed lines marked ZAHB (Zero age horizontal branch) are loci joining models with the same initial helium core mass that have just begun to burn helium at the center under nondegenerate conditions. Models beginning along each ZAHB differ only total mass (given in solar units below each initial model).

Evolution along each track proceeds in the direction of the arrows. The time for evolution between two adjacent tick marks is 5×10^6 yr.

Note that within two segments of each track (solid portions) the rate of motion along the track is markedly slower than is the rate of motion elsewhere (dashed portions). In the first segment, models are burning helium at the center and hydrogen in a shell. In the second segment, models are burning helium in one shell and hydrogen in another.

The region containing all models that begin on a common ZAHB and evolve slowly while burning helium at the center forms what we may call the HB (horizontal branch). In both the $Z = 10^{-3}$ and $Z = 10^{-4}$ cases, the HB has a luminosity width of about 0.2 mag.

The region containing all DSS models that evolve slowly (relative to the evolution rate in immediately preceding and following phases) forms another band that is above and parallel to the horizontal branch until it reaches the main giant branch. We shall call the "horizontal" portion of this band the AHB (above horizontal branch). In the $Z = 10^{-3}$ case the AHB has a luminosity width of about 0.5 mag and, at any color, its center lies above the HB by about 0.9 mag. In the $Z = 10^{-4}$ case, the AHB has a width of about 0.2 mag and its center lies about 0.6 mag above the HB.

Note that within both the HB and the AHB, less massive stars are on the average bluer than more massive stars. Further, at any chosen color, the more luminous a star is within each band, the less massive it is.

The tracks of all DSS stars eventually approach the giant branch and rise along it asymptotically. Thus the AHB merges smoothly into what may be called the AGB (asymptotic giant branch). Since all DSS tracks approach a common track at high enough luminosity, the AGB should be funnel-shaped.

From the model tracks in Fig. 2 we may argue that, in any cluster, the distribution of stars within the AHB and AGB should be related to the distribution of stars along the HB. A continuous distribution of stars along any segment of the HB ought to be reflected in a continuous distribution along a corresponding segment of the AHB and AGB. Because of the different relationship between stellar mass and mean T_e on the two branches, the distribution of stars on the AHB will be characteristically displaced and distorted relative to the distribution on the HB. For example, the bluest stars on the AHB should be considerably redder than the bluest stars on the HB.

The model results suggest that, if it were not for irregularities in the original luminosity function or in the extent of the mass loss which leads to the actual mass distribution of stars in advanced stages, the whole region defined by DSS models should be as continuously populated as is the horizontal branch (when displacement and distortion are taken properly into account). In particular, we would not expect the density of stars in the variable strip to be greater than just to the red of the strip. However, since *all* DSS models eventually become a part of the AGB, one might expect the AGB to stand out among the features above the HB and to the left of the main giant branch.

IV. Comparison Between Theory and Observation

On comparing our theoretical results with the properties of our program stars, we note first that no DSS models become bright enough and blue enough at the same time to account for the brightest blue cluster stars such as VZ 1128 and M10 I, 33. Moreover, during both the HB and AHB phases of model evolution, no formal mixing process of any sort occurs that would lead to the unusual surface abundances exhibited by these brightest, blue stars.

We suggest that the brightest, blue stars are the progeny of the double-shell source stars on the AGB. The recent calculations of Rose and Smith (1970) suggest that stars of mass $M < 1\ M_{\odot}$ will cross the region of the HR diagram occupied by planetary nebula central stars and by VZ 1128 and M10 I, 33 after undergoing a large number of thermal "pulses" on the AGB. The thermal instabilities developed during the double shell source phase may trigger ejection of a planetary nebula shell and/or large scale mixing. Thus, the unusual surface abundances of the brightest blue stars may be a consequence of mixing during the last brief stages of DSS evolution.

Turning now to our other program stars, we note again that, in each cluster, they lie above the HB by about 0.8 to 1.5 mag and are at least 0.15 mag redder than the blue end of the HB. This is, of course, in excellent qualitative agreement with the theoretical results which show that low mass DSS stars should lie above the HB in a band whose blue edge is redder than the blue edge of the HB.

We have demonstrated (see Fig. 2) that the luminosity width (W) of the band and its height (H) above the HB should depend on the metal abundance. Both W and H increase with increasing Z. Comparing with the observed heights above the HB (see Table 2),

we might conclude that, if $Y \sim 0.3$ is a good choice, then $Z \lesssim 10^{-3}$ is appropriate for most of our program stars. Both W and H depend also on Y. It is conceivable that this dependence might eventually be made use of as an added check on estimates of Y.

It is pertinent to ask if the relative numbers of AHB, AGB, and HB stars are in proportion to the corresponding theoretical lifetimes. We choose two examples, M13 and M92, and use Arp's (1955) early study as being relatively free from selection effects. The stars in M13 from Arp's Table III are shown in Fig. 1 with the various regions of interest marked. We estimate 5 AHB stars between -0.1 mag and $+0.5$ mag in color index (corresponding to ~ 0.05 to 0.65 in $[B-V]$). If we choose $Y = 0.3$ and $Z \sim 10^{-3}$ as appropriate for M13, then using Fig. 2, we find that AHB stars should originate from that region of the HB bluer than $\log T_e \sim 4.00$ to 4.05 (roughly C.I. $\lesssim 0.2$ after reddening corrections). There are between 7 and 13 stars in this region. The ratio of lifetimes for stars in the two selected regions should be (within a factor of two) between 7/5 and 13/5. The model tracks suggest a lifetime ratio of about 2 to 1 or 3 to 2, in satisfactory accord with the observed ratios.

Of the total of 83 HB stars, 70 should in time feed the AGB, the remainder feeding the AHB. In Fig. 8 of Arp's work there are about 15 stars that are clearly distinguishable from the main giant branch. Beyond the point where the two branches merge are about 40 stars. If we assign half of these to the AGB, as suggested by the relative lifetimes, we have a total of about 35 AGB stars. The ratio of theoretical lifetimes is in accord with the 70/35 ratio of "red" HB stars to AGB stars that we have just estimated.

We analyse M92 in the same way, but this time assume that the $Z = 10^{-4}$ tracks in Fig. 2 are more appropriate. In order to populate an AHB, stars must first reach a point on the HB to the blue of $\log T_e \sim 4.05$. In M92 there are no HB stars as blue as this. Since there are also no AHB stars, theory and observation agree again. We estimate 30 AGB stars and 50 HB stars. The ratio of 50/30 is in satisfactory accord with the ratio of theoretical lifetimes in the HB and DSS phases.

Of course the number of stars in the various regions in M13 and M92 is small and consequently the comparison between theory and observation cannot be exact. Nevertheless, we find the general agreement to be quite gratifying.

In closing we comment on several observational programs worth pursuing. (1) An effort should be made to determine the distribution of stars between the variable strip and the AGB. The theory suggests that there should be a smooth gradient in star density in this region. The fact that the density of stars in the variable strip appears, in several published diagrams, to be higher than to either side of the strip is possibly a result of selection. (2) An effort should be made to compare the rates at which the periods of BL Her and W Virginis stars change with time. We suggest that BL Her stars are DSS stars evolving slowly through the AHB. Low luminosity W Virginis stars may be low mass stars originating and spending most of their DSS life near the blue edge of the AHB. They may be passing through the variable strip rapidly relative to the rate at which the BL Her stars pass. Finally, those W Virginis stars that are more than $1^1{}_2$ to 2 mag brighter than RR Lyrae stars are probably not in the DSS phase. They may be evolving rapidly away from the giant branch into the region occupied by VZ 1128 stars. If so, they may show, in the aggregate, a distinct bias toward decreasing periods. (3) Schwarzschild and Härm (1967) and Sanders (1967) have suggested that large quantities of s-process elements may be formed during one of the thermal pulses of a DSS star. Consequently, we should seek observational evidence for s-process element enhancement in the cool DSS stars or their descendents. The luminous W Vir stars and the asymptotic giant branch stars are prime candidates for such a search. Furthermore, we might expect in these stars evidence for C, N and O enrichment as a consequence of transporting material from the core or from the hydrogen burning shell to the surface of the star.

Conclusions

We have established that a large majority of stars observed to lie above the HB in some globular clusters are probably members. Using the results of model calculations, we suggest that most of these stars are in the DSS phase and most have probably lost considerable mass before arriving on the horizontal branch. The most luminous blue stars may have already passed through the double-shell source phase and have started their rapid evolution toward white dwarf status. Stars in this most luminous group appear from their spectra to have an unusual surface composition which may arise from large-scale mixing during the double-shell source phase.

References

Allen, C. W. 1963, Astrophysical Quantities (2nd edition) Athlone Press, London.

Arp, H. C. 1955, *A. J.* **60**, 317.

Greenstein, J. L., Münch, G. 1966, *Ap. J.* **146**, 618.

Greenstein, G. S., Truran, J. W., Cameron, A. G. W. 1967, *Nature* **213**, 871.

Kadla, P. 1966, *Publ. Pulkova Obs.* **5**, 93.

Kinman, T. D. 1959, *M.N.R.A.S.* **119**, 538.

Iben, I. Jr., Rood, R. T. 1970a, *Ap. J.* **159**, 605.

Iben, I. Jr., Rood, R. T. 1970b, *Ap. J.* **161** (in press).

Iben, I. Jr., Rood, R. T. 1969, *Nature* **223**, 933.

Iben, I. Jr., Rood, R. T., Strom, K. M., Strom, S. 1969, *Nature* **224**, 1006.

Mayall, N. U. 1946, *Ap. J.* **104**, 290.

Newell, E. B. 1970, *Ap. J.* **159**, 443.

Przbylski, A. 1969, *M.N.R.A.S.* **146**, 71.

Rose, W. K., Smith, R. L. 1970, *Ap. J.* **159**, 903.

Sandage, A. R., Smith, L. L. 1966, *Ap. J.* **144**, 886.

Sanders, R. H. 1967, *Ap. J.* **150**, 971.

Sargent, W. L. W., Searle, L. 1966, *Ap. J.* **145**, 652.

Schwarzschild, M., Härm, R. 1967, *Ap. J.* **150**, 961.

Searle, L., Rodgers, A. W. 1966, *Ap. J.* **143**, 809.

Stoeckley, R., Greenstein, J. L. 1968, *Ap. J.* **154**, 909.

Strom, S. E., Strom, K. M. 1970, *Ap. J.* **159**, 195.

Traving, G. 1962, *Ap. J.* **135**, 439.

S. E. Strom
K. M. Strom
Earth and Space Sciences Dept.
State University of New York at Stony Brook
Stony Brook, L.I., New York, USA, 11790

R. T. Rood
I. Iben, Jr.
Massachusetts Institute of Technology
Cambridge, Mass. USA, 02139

Fig. 1. The color-magnitude diagram for M13 as given by Arp (1955). The program stars Barnard 29, (II, 48), (IV, 52), L326, and L526 are labelled. L656 is not shown. The stars with Ludendorff numbers are not in the region of Arp's study and are not included in the number counts used in the discussion of lifetimes. Stars are classified into the following groups: the main horizontal branch (HB), the above horizontal branch (AHB), the asymptotic giant branch (AGB), the red giant branch (RGB), and those stars similar to Von Zeipel 1128

Fig. 2. Evolutionary tracks for the core helium burning and double shell source phases of models with envelope compositions $Y = 0.30$, $Z = 10^{-3}$ and 10^{-4}. The dashed lines labelled ZAHB are the loci of initial horizontal branch models with a constant core mass ($0.475\ M_\odot$). Total model mass is indicated below the location of each initial model. Rapid phases of evolution are represented by the dashed portions of the tracks. The time interval between adjacent tick marks on any track is 5×10^5 years

Table 2. *Location of program stars relative to cluster horizontal branches*

Cluster	Star	V	ΔV_{AHB}	$\Delta(B-V)$
M2	I, 59	15.2	−1.0	+0.2
	IV, 96	14.6	−1.4	+0.3
M3	VZ 1128	15.0	−3.0[a]:	−0.4
M10	I, 32	13.8	−1.5	+0.15
	I, 33	13.5	−2.1	−0.1:
	I, 34	14.0	−1.0	−0.3
M13	Barnard 29	13.0	−3.0	−0.05
	II, 48	13.7	−1.2	−0.35
	IV, 52	13.9	−1.0	−0.40
	L326	13.9	+1.0:	−0.4:
	L526	13.8	−0.8:	−0.6:
	L656	14.5	?	?
M15	IV, 50	15.1	−0.8	−0.3:
NGC 6712	C26	13.1	−3.6	−0.1

[a] Based on eye extrapolation of blue end of horizontal branch.

Table 3. *Description of stellar spectra*

Star	Description
M2 I, 59	very sharp, fairly weak K line; H lines have sharp cores; possible weak Mg II λ4481 A0 – A2 III
IV, 96	moderately weak K line; H lines moderately sharp; no λ4481 A0 – A2 III
M3 VZ 1128	He I, He II spectrum; lines of O II, N III, possibly Si IV O9 V (see Strom and Strom, 1970)
M10 I, 32	very weak K line; no λ4481; no He lines; λ4077 possible A0 III
I, 33	Helium I spectrum pronounced (B5); 3995 possible; possible 4116 – 4121 blend; O II 4349; Si III 4552; very unusual spectrum; B3 – B5 Ib ?
I, 34	sharp H lines; fairly strong K line; No λ4481 wk. line A7 – F0 III based mainly on K line type
M13 Barnard 29	see (Stoeckley and Greenstein (1968) and Traving (1962)) H and He spectrum; B3
II, 48	weak K line. H lines fairly broad; possible weak 4471; possible broad feature at λ4077; possible λ4215; possibly unusual spectrum; ~~however,~~ background from image tube strong from He and Ca type: B9 – A0 V
IV, 52	sharp K line present; fairly broad H lines; no λ4481; blend at 4272, λ4045 possibly present; wk. line A3 – A5 III – V.
L326	weak K line. sharp cores to H lines; 4481 weak possibly present; A3 III (?)
L526	*very* sharp H lines; possible λ4045, λ4183, λ4481; looks like A5 Ib, but could very well be weak line star of later type
L626	looks like early K-giant; possibly composite spectrum (?) since it has strong blue continuum and unusual H line strength in blue; needs further study
M15 IV, 50	*possible* weak He K λ4471 although neither λ4026 or λ3820 is present; very weak features present at λ4045, λ4183 B9 – A0 III – V
NGC 6712, C26	He I, λ3820, λ4026 and λ4471 weak but present; K [illegible] weak λ4481; e [illegible]

Table 4. *Helium line strengths in* M 10 I, 33

Line	M 10 I, 33	ι Her
3820	1.0 Å	
4009	0.6	0.60
4026	1.3	1.35
4120	0.35	0.22
4143	0.75	0.80
4388	0.65	0.77
4471	0.85	1.24
4713	0.6	

Table 1. *Program star radial velocities*

Cluster	Star[a]	N	$<V>$	V_{cl} (Mayall)	V_{cl} (Kinman)	Member?	Comment
M2	I, 59	2	— 21	— 3 km/s	—12 km/s	yes	
	IV, 96	1	— 15			yes	
M3	Von Zeipel 1128	1	—157	—150		yes	
M10	I, 32	2	+ 36	+ 73		possible	low wt.
	I, 33	3	+ 51			yes	
	I, 34	1	+ 55			possible	low wt.
M13	Barnard 29	1	—225	—228		yes	PM
	II, 48	2	—239			yes	PM
	IV, 52	2	—214			yes	PM
	L 236	2	—235			yes	PM
	L 526	3	—252			yes	PM
	L 656	1	—249			yes	PM
M15	IV, 50	1	—117	—114	— 98	yes	
NGC 6712	C 26	2	—152	—131		yes	

[a] The designations (Roman numeral, arabic numeral) are from Arp (1955).
L denotes Ludendorff number (see Kadla, 1964).
References to Barnard 29 and VZ 1128 may be found respectively in Stoeckley and Greenstein (1968) and Strom and Strom (1970).
PM denotes membership on the basis of Kadla's (1964) survey.